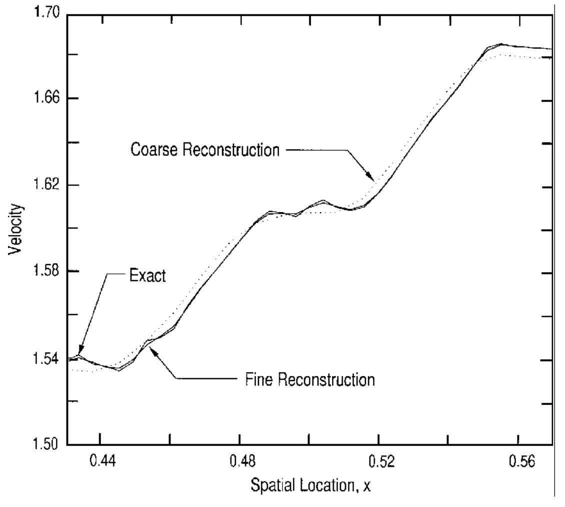

Fig. 2 Effects of spatial underresolution.

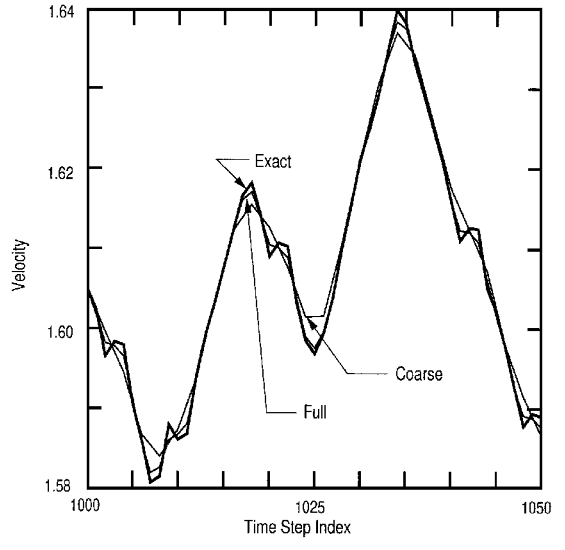

Fig. 3 Effects of temporal underresolution.

Conclusions

In this study, we have investigated effects of resolution of the measured passive scalar field on accuracy of reconstructed velocity in the context of chaotic flows using a one-dimensional version of the method proposed by Pearlstein and Carpenter.[1] The results of our numerical tests lead us to conclude that it should be possible to experimentally determine time-dependent, turbulent flowfields from measurements of a passive scalar that is convected by the flow if measurements of scalar data are sufficiently accurate and well resolved. The resolvable accuracy of a velocity field obtained via this technique is limited by the resolution of the scalar measurements, as would be expected, thus implying that very high-resolution scalar measurements would be needed to completely reconstruct fine-scale features of high-Reynolds-number turbulence. On the other hand, we also found that large-scale flow features can be extracted even from underresolved scalar data.

We comment in closing that the approaches presented in Refs. 1 and 2, along with the results presented here (and in Refs. 3 and 4), provide a hint of the significant potential of techniques from computational fluid dynamics in extracting far more detailed and accurate information from laboratory flowfield experimental data than has heretofore been possible. Moreover, as mentioned in Ref. 4, there appears to be a significant potential for applying such methods to real-time flow control problems. Our findings in the present study concerning the ability to fairly accurately reconstruct large-scale features of flowfields using underresolved scalar data could be very significant in this context because of the greatly improved computational efficiency associated with low-resolution calculations.

Acknowledgments

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Torsional Instability of Moderately Thick Composite Cylindrical Shells by Various Shell Theories

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Introduction

BUCKLING analyses for torsion started with the work of Donnell.[1] Numerous studies followed with the emphasis on different considerations. For thin shells of homogeneous material, classical shell theory predicts in-plane stresses, deformations, and consequently buckling loads comparable to those given by three-dimensional elasticity. A great deal of interest for the continuous development of refined shell theories has been manifested in recent years in the literature. The greatest stimulus for such theories arises from the fact that classical theory in terms of its basic assumptions comes into conflict with the real behavior of the moderately thick shells made of composite materials largely used in the modern technology. Thus, in contrast to one of the basic assumptions, implying an infinite rigidity in transverse shear, the new composite materials exhibit a finite rigidity in transverse shear. This property requires the incorporation of transverse shear deformation effect.

The present work deals with the development of the kinematic relations, equilibrium equations, buckling equations, and related boundary conditions for laminated, cylindrical, moderately thick shells, including the effect of transverse shear. Buckling applications are presented for moderately thick, torsionally loaded, laminated,

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cylindrical shell with fixed ends. Results are generated for three theories (classical and first-order and higher-order shear deformation) based on Donnell-type and Sanders[2]-type kinematics relations. The results are compared to the ones predicted by the higher-order theory (which leads to predictions of displacements and stresses, close to the predictions of the more accurate three-dimensional elasticity analyses) to determine the range of applicability and necessity of each theory for the analysis of moderately thick, torsionally loaded, cylindrical shells.

Mathematical Formulation and Solution

The cylindrical shell is assumed to be relatively thick and geometrically perfect and have a laminated construction, which is symmetric with respect to the midsurface. The material behavior is linearly elastic, the laminae are orthotropic, and the loading is torsion. From the exact nonlinear theory of elasticity for strains, and by performing a power series expansion and retaining terms to second degree, the expressions for the strains are obtained (for details, see Ref. 3).

A general cubic variation in the z coordinate is assumed for u (axial displacement) and v (circumferential displacement), whereas w (along the thickness displacement) is independent of z:

$$u(x, y, z) = \bar{u}(x, y) + z\Psi_x(x, y) + z^2\xi_x(x, y) + z^3\zeta_x(x, y)$$

$$v(x, y, z) = \bar{v}(x, y) + z\Psi_y(x, y) + z^2\xi_y(x, y) + z^3\zeta_y(x, y) \quad (1)$$

$$w(x, y, z) = \bar{w}(x, y)$$

where $\bar{u}(x, y)$, $\bar{v}(x, y)$, and $\bar{w}(x, y)$ are reference surface displacements and Ψ_x, ξ_x, ζ_x, Ψ_y, ξ_y, and ζ_y are functions of position on the reference surface (x, y). For the purpose of this work, concentration on problems where the rotations $\{\bar{W}_{,x}$ and $\bar{W}_{,y} - [\delta_1 \bar{v}/(R+z)]\}$ are moderately small is made.[3,4] Note that if $\delta_1 = 0$, then Donnell-type relations are obtained and if $\delta_1 = 1$, then Sanders-type relations are obtained. Thus, products containing these terms are retained, and the remaining products are neglected as small by comparison. Substitution of the displacement relations into the strain expressions and the knowledge of zero shear tractions on the upper and lower surface lead to the final kinematic relations.

Use of the principle of the stationary value of the total potential yields the equilibrium equations and associated boundary conditions. The kinematic relations used to obtain the equilibrium equations are of the Donnell type (setting $\delta_1 = 0$). The shell is free of initial geometric imperfections, the laminate is symmetric with respect to the midsurface, and the only loading is torsion $\bar{N}_{x\theta}$. These conditions allow the existence of a prebuckling state, and bifurcation buckling is possible. The primary state solution is found to be the following:

$$\bar{u}^p = Ax$$

$$\bar{v}^p = Bx + C\theta \quad (2)$$

$$\bar{w}^p = \Psi_x^p = \Psi_y^p = 0$$

where the constants A, B, and C are found in term of the applied load $\bar{N}_{x\theta}$ and the shell stiffness parameters. Once the prebuckling state solution is known, the perturbation technique can be applied to produce the buckling equations and the related boundary conditions. The buckling equations can be derived by assuming that we can pass from the primary to the buckled state through extremely small additional parameters so that linearization is possible (for more details, see Ref. 3). The justification for using Donnell-type relations for equilibrium and then Sanders-type relations (or Donnell type) for buckling is that the rotation $v/(R+z)$ is more important in the buckled state.

A solution in the form of double trigonometric series in the x and the y (or θ) directions is employed for all five variables: $\bar{u}$, $\bar{v}$, $\bar{w}$, Ψ_x, and Ψ_y. (This is after using the conditions that $\tau_{\theta z} = \tau_{xz} = 0$ on $z = \pm h/2$, which leads to expressing ξ_x, ξ_y, ζ_x, and ζ_y in terms of the remaining unknown functions $\bar{u}$, $\bar{v}$, $\bar{w}$, Ψ_x, and Ψ_y.) Each term

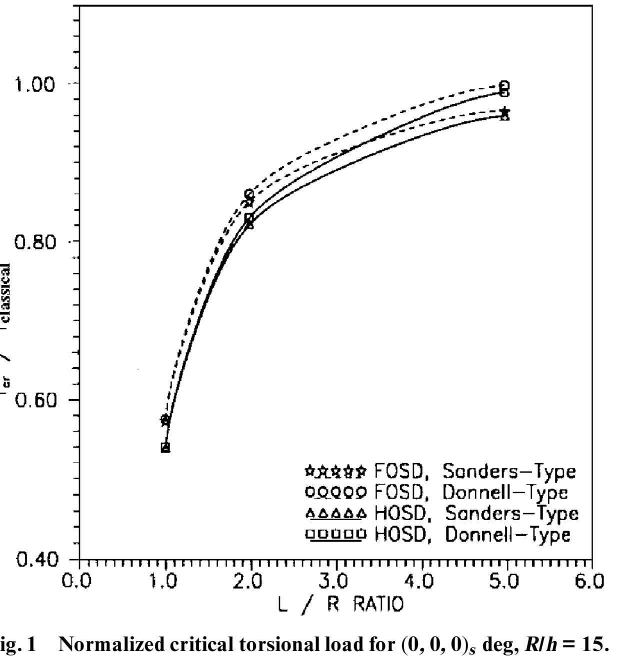

Fig. 1 Normalized critical torsional load for $(0, 0, 0)_s$ deg, R/h = 15.

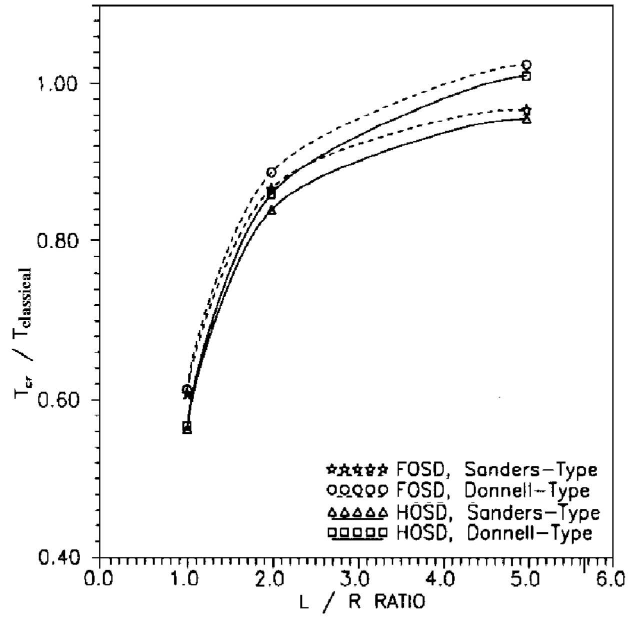

Fig. 2 Normalized critical torsional load for $(0, 90, 0)_s$ deg, R/h = 15.

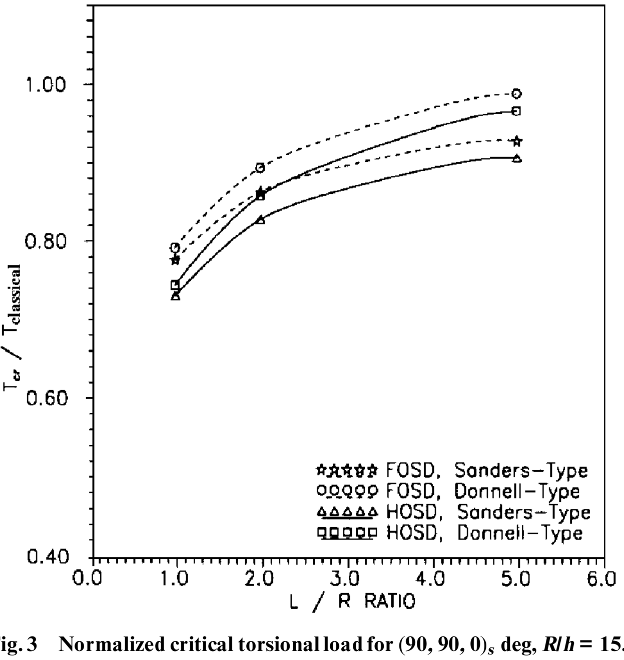

Fig. 3 Normalized critical torsional load for $(90, 90, 0)_s$ deg, R/h = 15.

in the series satisfies the boundary conditions. Substitution into the Galerkin integrals leads to a system of linear algebraic equations in the series coefficients. For a nontrivial solution to exist, the determinant of the coefficients must vanish. This determinant is the characteristic equation. Convergence study has been performed on the number of terms (m) in the series solution and found that $m = 30$ yields convergent state for all geometries considered. Minimum critical load was obtained by iteration on the number of circumferential waves n.

The material considered in this analysis is graphite/epoxy. The cylinder radius is $R = 0.1905$ m, and the length is varied such that $L/R = 1$, 2, and 5. The total thickness values of $h = 0.001905$, 0.003175, 0.00635, and 0.0127 m are considered.

Numerical Results and Discussion

To establish a confidence factor of the developed equations (considering the fact that Donnell-type relations were employed in deriving the equilibrium equation and then Sanders-type relations were employed in the buckling equations), a comparison is made to some results from the analysis by Simitses and Han.[5] Results predicted by classical theory (CL) for CC-4 type boundary conditions in the present analysis are compared to those of Simitses and Han[5] for SS-4-type boundary conditions, in which the typical Sanders-type analysis was employed. The results are of the usual trend; as the cylinder becomes long the values of the critical loads for the two different boundary conditions converge to each other.[6]

Results are presented for cylinders under torsion for graphite/epoxy material and nine stacking sequences. Three theories are employed in the generation of results: CL theory, first-order shear deformation theory (FOSD), and higher-order shear deformation theory (HOSD). Moreover, one set of results is obtained by employing Sanders-type kinematic relations ($\delta_1 = 1$) and one by employing Donnell-type kinematic relations ($\delta_1 = 0$). The intention here is to identify the parameters that affect the accuracy of critical torsional load established through Donnell-type kinematic relations including the effect of transverse shear, by comparing them to those established by Sanders-type kinematic relations, which also include the effect of transverse shear. This objective is achieved by varying the cylinder length and the shell thickness in order to cover the range of practical interest. Note that to accommodate all thicknesses used, each ply in the stacking sequences consists of several similar plies. Therefore, in reality $(0, 0, 0)_s$ deg means $(0_n, 0_n, 0_n)_s$ deg, where n can be any number.

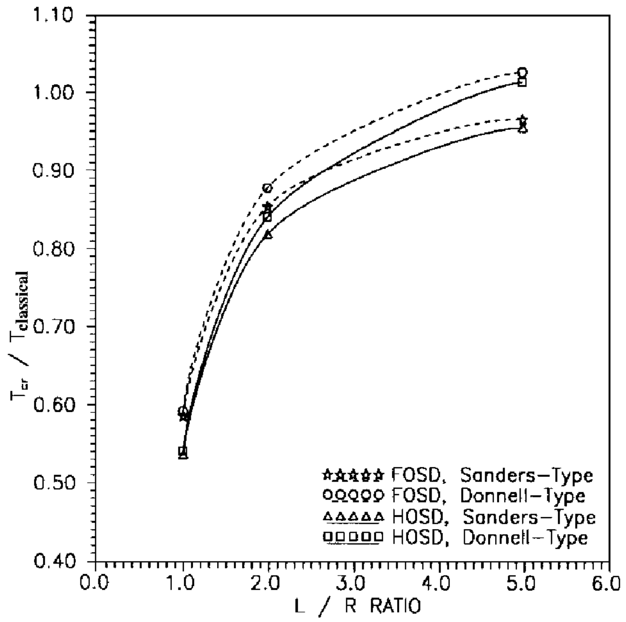

Fig. 4 Normalized critical torsional load for $(0, 90, 90)_s$ deg, $R/h = 15$.

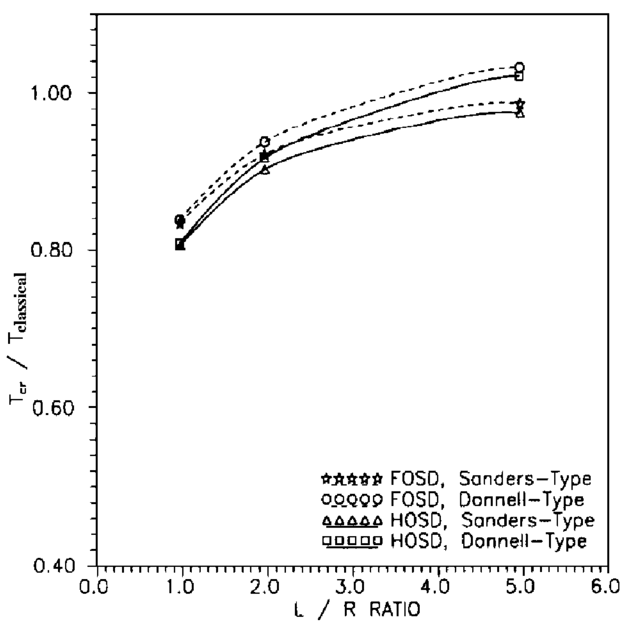

Fig. 5 Normalized critical torsional load for $(-45, -45, 45)_s$ deg, $R/h = 15$.

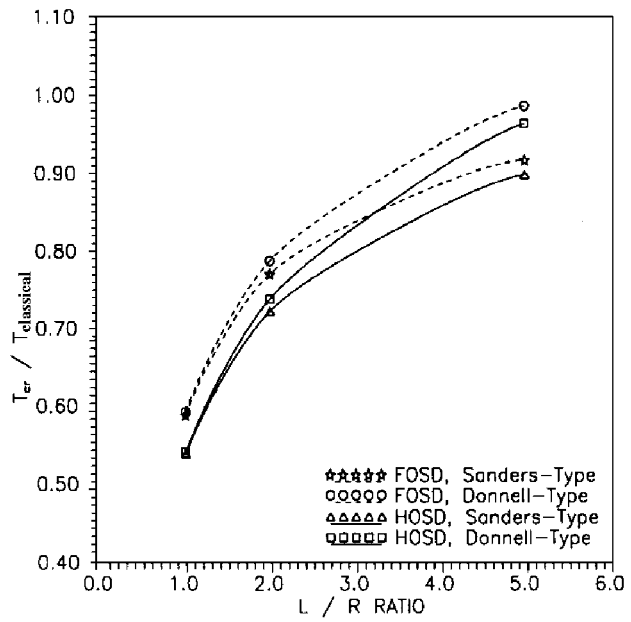

Fig. 6 Normalized critical torsional load for $(45, -45, -45)_s$ deg, $R/h = 15$.

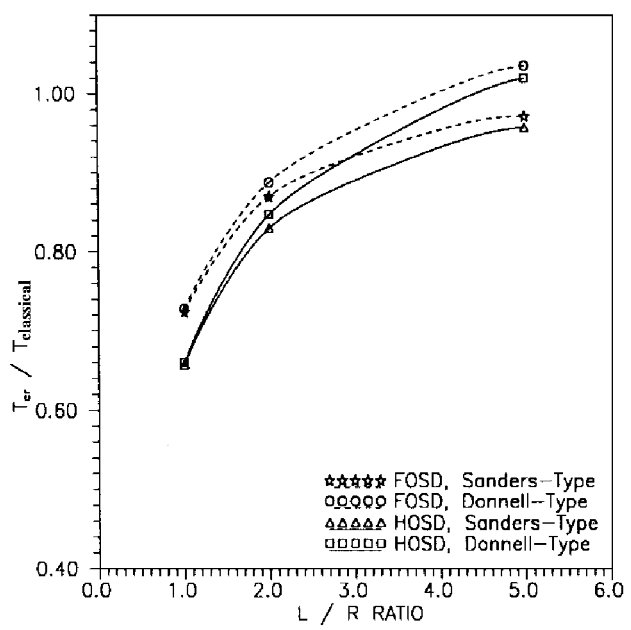

Fig. 7 Normalized critical torsional load for $(-45, 45, -45)_s$ deg, $R/h = 15$.

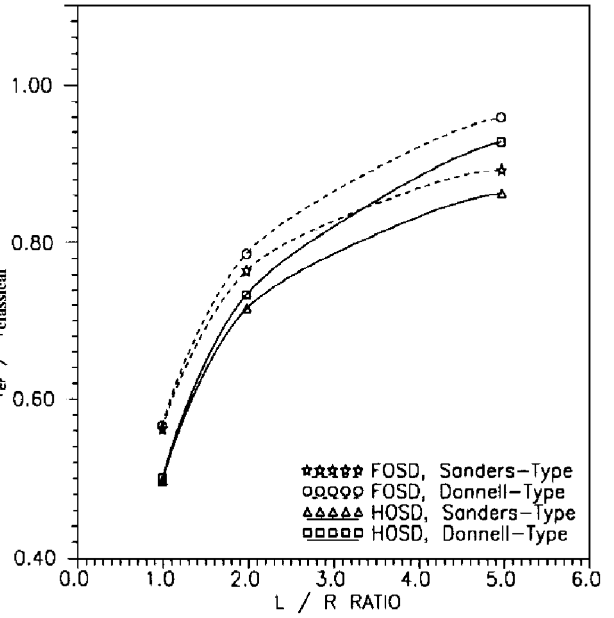

Fig. 8 Normalized critical torsional load for (45, 45, _45)$_s$ deg, R/h = 15.

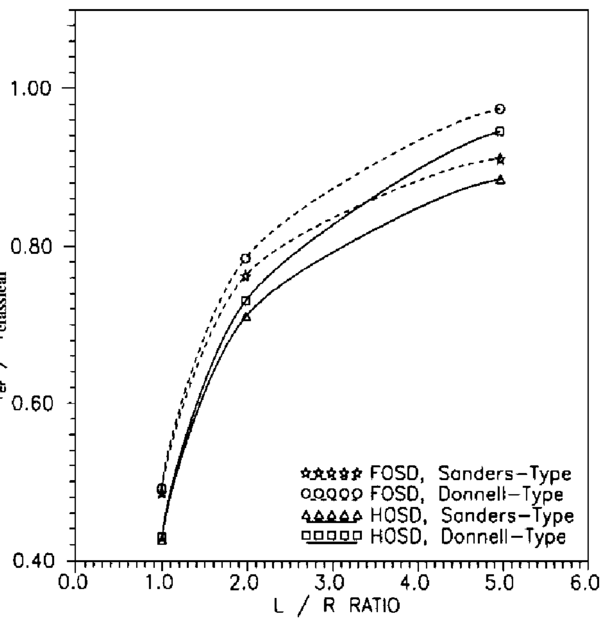

Fig. 9 Normalized critical torsional load for (30, 30, _60)$_s$ deg, R/h = 15.

Figures 1–9 show the critical torsional load for the shear deformation theories normalized with respect to the values obtained by the classical theory. In general, critical loads obtained by employing Donnell-type relations predicted higher values than those obtained by employing Sanders-type relations. This observation is true for all R/h ratios and L/R ratios. The difference between the two sets is small for small L/R and R/h ratios and becomes more evident as the shell becomes thicker and longer. The discrepancy between critical loads obtained from the two different approximations (Sanders and Donnell) is primarily affected by L/R ratio and to lesser degree by R/h ratio. When n (the number of full waves) is > 3, the two sets yield almost the same critical load (within 1%), but for $n \leq 3$ the computed difference can be as large as 8%. For all of the generated results (all R/h and L/R ratios), the critical loads were within the range of 0.5–8% above the ones predicted by employing Sanders-type relations. This is attributed to the fact that the rotation $[\bar{v}/(R+z)]$ has more effect when the number of circumferential waves n is < 3. Note that the effect of L/R ratio is the same for all three theories and including the effect of shear deformation has no effect on the behavior of the critical loads obtained by Donnell-type relations.

Acknowledgment

Computing support was provided by the Ohio Supercomputer Center and it is gratefully acknowledged.

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Final Solution of Duffing Equation of Mixed Parity

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Introduction

HIAMANG and Mickens[1] have examined the periodic solution of the nonlinear second-order differential equation of motion,

$$\ddot{x} + x^3 = 0 \tag{1}$$

with the initial conditions

$$x = x_0, \qquad \dot{x} = 0 \quad \text{at} \quad \tau = 0 \tag{2}$$

and the corresponding first-order differential-equation energy relation, applying the method of harmonic balance with lower-order harmonics. The overdots denote differentiation with respect to time τ. The values of angular frequency determined from the equation of motion and the energy relation differ by approximately 20%. The periodic solution from the equation of motion is found to be an excellent first approximation to the exact solution. On the basis of these observations, they concluded that the application of harmonic balance, in the lowest-order approximation, to the energy relation is not particularly helpful in obtaining the periodic solution and suggested using only the equation of motion. Strictly speaking, the first-order differential-equation energy relation is derived from the second-order differential equation of motion, and so, one should expect the same exact/approximate result from these equations. Reinvestigation of this problem by Narayana Murty and Nageswara Rao[2] reveals that the solution from the Hiamang–Mickens[1] energy equation does not satisfy the initial conditions exactly. The inclusion of higher-order harmonics in the method of balance gives better

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